The Risk of Miscomputing the Value at Risk

O Risco de Calcular mal o Valor em Risco

Carlos Rodríguez

Doctorado en Ingeniería de Sistemas por la Universidad Nacional Autónoma de México / Facultad de Ingeniería
Institución: Universidad Nacional Autónoma de México /
Lugar de trabajo: IIMAS-UNAM
Dirección de la Institución: Av. Universidad 3000, C.P. 04510, Ciudad de México
Correo electrónico: crc@unam.mx

ABSTRACT
This article explores how the VaR (Value at Risk), which is the most popular financial risk metric is commonly computed and used. A huge amount of misunderstanding of this technique still persists in the financial industry, from what it is, what it is for, how it is used, and even who should use it. Although VaR is no longer new, in many organizations, both in academia and in industry, it is still implemented as it was conceived in the 1990s as a first effort to quantify Financial Risk. Given that VaR is strongly supported by Modern Portfolio Theory, and that this in turn was elaborated under the assumption that the oscillations of financial signals behave under a Normal Probability Distribution, this is how it is still computed in many organizations that apply it to control the trading of Spot and Derivatives financial assets. In this article, the use of the Scaled Student’s t Distribution is discussed as a better option for modeling the time series of financial returns. The returns modeled with this distribution, in turn, allow the Value at Risk to be computed with greater precision. Additionally, with this distribution, the risk metric created as a great improvement to VaR can be computed: The Expected Shortfall (ES), also known as the Conditional VaR (CVaR). To demonstrate that the Scaled Student’s t distribution is better for modeling financial signals on stock returns, and therefore for the computation of VaR and ES, three graphs of different probability distributions are generated and overlapped: The empirical distribution, the normal distribution and the Scaled Student’s t distribution computed with the Maximum Likelihood Estimation (MLE) technique. This is done for each of the six stocks analyzed in this study: The FAANG, plus the one recently added to the S&P 500: Tesla.

Keywords: Value at Risk, Expected Shortfall, Probability Distributions in Finance.
por sua vez, permitem que o Value at Risk seja calculado com maior precisão. Além disso, com essa distribuição, pode-se calcular a métrica de risco criada como uma grande melhoria para o VaR: The Expected Shortfall (ES), também conhecido como VaR Condicional (CVaR).
Para demonstrar que a distribuição t de Student em escala é melhor para modelar sinais financeiros nos retornos de ações e, portanto, para o cálculo de VaR e ES, três gráficos de distribuições de probabilidade diferentes são gerados e sobrepostos: A distribuição empírica, a distribuição Normal e a distribuição t de Student em escala, calculadas com a técnica de estimativa de máxima verossimilhança (Maximum Likelihood Estimation).
Isso é feito para cada uma das seis ações analisadas neste estudo: O FAANG (Facebook, Apple, Amazon, Netflix, and Google), mais aquele recentemente adicionado ao S&P 500: Tesla.

**Palavras-chave:** valor em risco, déficit esperado, distribuições de probabilidade em finanças.

### 1 INTRODUCTION

The problem of risk measurement is one of the ancient ones in statistics, economics and finance. Financial risk management has been a concern of financial regulators and executives for a long time as well. Retrospective analysis has found some concepts such as value at risk in this story. Dickson Leavens (1945) did not explicitly identify a VaR metric, but he mentioned repeatedly the “spread between probable losses and gains.” He seems to have considered the standard deviation of portfolio market value.

The fact is that, in order to control and keep out of possible financial risk, managers and investors throughout the world have paid increasing attention to the research on financial risk. The first and the most important step in researching financial risk is risk measurement. Markowitz who won the Nobel prize in economics in 1990 proposed measuring return and risk with mean and variance, respectively, in his portfolio theory, and it is the beginning of quantitative research on financial risk.

Till Guldimann (2000), the creator of RiskMetrics, suggests that the name “value-at-risk” originated within JP Morgan prior to 1985, in his article “The story of RiskMetrics” he wrote:

… we learned that “fully hedged” in a bank with fully matched funding can have two meanings. We could either invest the Bank’s net equity in long bonds and generate stable interest earnings, or we could invest it in Fed funds and keep the market value constant. We decided to focus on value and assume a target duration investors assigned to the bank’s equity. Thus value-at-risk was born.

However, VaR did not emerge as a concept by his own until the late 1980s with the proliferation of new instruments and opportunities for leverage. New instruments and new forms of transactions started to offered leverage. Commodity leasing, securities lending and short sales are leveraged transactions. All of these either did not exist or had limited use prior to 1970 (Jorion, 2006).

As leverage proliferated, trading organizations were in need of new ways to manage risk taking. In turn, this motivated a need for new measures of financial risk. The traditional risk metrics of financial accounting were ineffective, especially when applied to derivatives markets. Exposure metrics such as duration, convexity, delta, gamma, and vega were widely adopted, but were primarily of tactical and not
strategic value. Trading organizations appeared to be in a Babel Tower, with each trading agency adopting risk metrics suitable for its own transactions (Jorion, 2006).

But what grabbed the world’s attention to the risk metrics topic, was the dramatic failure of Britain’s Barings PLC in February 1995. Nick Leeson, a young trader, later known as the Rogue Trader, based at its Singapore office, lost USD 1,400MM from unauthorized Nikkei 225 futures and options positions.

Barings had been founded in 1762. It had financed Britain’s participation in the Napoleonic wars. It had financed America’s Louisiana purchase and construction of the Erie Canal. Following its collapse, Barings was sold to Dutch bank ING for the price of one GBP. (Holton, 2002, Jorion, 2006).

In 1996, most probably aroused for the Baring affair, the Basel Committee approved the limited use of proprietary value-at-risk measures for calculating the market risk component of bank capital requirements. In this and other ways, regulatory initiatives helped motivate the development of proprietary value-at-risk measures (Holton, 2002).

From then now, it is well known that the Basel Accords oblige banks to use two methods to measure and limit risk – value at risk and expected shortfall – but research shows that in many cases, these are still insufficient to curtail the behaviour of rogue traders (Brigo and Armstrong, 2018).

VaR is a category of probabilistic measures of market risk. To understand this, consider a portfolio of shares. Its current market value is known, but its market value at some future time, say one day or one month in the future, is a random variable.

As a random variable, portfolio returns may draw a known probability distribution besides the empirical distribution. So, the VaR metric is a function of:
1. that Probability Distribution and
2. the Portfolio’s current market value.

Early VaR measures developed along this two parallel lines. One was portfolio theory, and the other was computation adequacy for capital assets oscillations. This paper focuses primarily upon the development of VaR measures in the context of adequacy computations for capital assets oscillations, which means making efforts to find a Probability Distribution that better fit the observed stock returns, and as such, developing models to better financial risk analysis.

THE PROBABILITY DISTRIBUTIONS IN FINANCE

Probability distributions has been used in the financial industry as a way to model and quantify the outcomes of financial interactions, from the start of market activity such as real time option pricing, to the end of day calculation of a portfolio’s Value-at-Risk (VaR). By assuming the distribution of a data, Financial Analysts can utilize the characteristics of the distribution to make predictions on outcomes. A commonly used probability distribution for these affairs is the Normal Distribution:
for its simplicity of having two easily identifiable parameters: mean and variance, and the widespread justified argument that most populations are distributed normally when sampled at large numbers (Ang, 2014).

However, in real life, it is very evident that financial phenomena such as stock returns do not behave under a Normal Distribution. This has been known since the conception of Modern Portfolio Theory, so efforts have been made to repair this enormous drawback. If the initial considerations are wrong, the whole analysis will produce wrong results.

Even knowing this great deficiency, a large number of financial organizations continue to carry out their Value at Risk analysis under this consideration, accepting that the results obtained have little impact on financial decisions, almost taking this analysis as an obligatory waste of time in the organization’s daily tasks.

Among the Probability Distributions used to solve the mentioned inconvenience, Student t-distributions:

\[ f(t) = \frac{\Gamma\left(\frac{\nu + 1}{2}\right)}{\sqrt{\nu \pi} \Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \]

are generally applied in financial studies as heavy-tailed substitute to the normal distribution. Therefore, distributions of logarithmic asset returns can often be fitted extremely well using Student t-distribution with \( \nu \) degree of freedom (Kumari and Tan, 2013).

GETTING FINANCIAL DATA FROM YAHOO FINANCE

For the purposes of this study, an investment portfolio is built with the FAANG assets, which have high influence in the S&P 500, Tesla is also added to the stocks being studied because it is intended to be the same or bigger influence in the S&P after its inclusion this December 21, 2020:

- FB (Facebook)
- AAPL (Apple Computing)
- AMZN (Amazon)
- NFLX (Netflix)
- GOOGL (Google)
- TSLA (Tesla)
A period of five years is taken, ending on the day this document is compiled. The following figures show graphs of Japanese candlesticks for each stock considered for a period from January 1, 2020 to the compilation day. Three Technical Analysis indicators are added, such as trading volume, Bollinger Bands and the MACD indicator. All these graphs show the bear market effect caused by the COVID-19 pandemic.

Figure 1. Facebook Candlesticks Chart.

Figure 2. Apple Candlesticks Chart.
Figure 3. Amazon Candlesticks Chart.

Figure 4. Netflix Candlesticks Chart.
Figure 5. Google Candlesticks Chart.

Figure 6. Tesla Candlesticks Chart.
LOGARITHMIC RETURNS OF THE ASSETS

When examining financial time series it is most common to study the returns rather than the actual raw asset prices. The reasons for analyzing the returns rather than the asset price are that they give us a scale-free assessment of the performance of the asset and that returns also have more attractive statistical properties for analysis. In addition, as discussed above, asset prices and the logs of the asset prices observed at any time frequency are not normally distributed (Quigley, 2008).

A continuously compounded return is defined as the natural logarithm of the gross simple return of the asset and is given by

\[ r_t = \ln(1 + R_t) \]
\[ = \ln \left( \frac{S_t}{S_{t-1}} \right) \]
\[ = \ln S_t - \ln S_{t-1} \]

This is the difference between the natural log of the assets price at time \( t \) and the natural log of its price at the previous step in time. Due to this definition \( r_t \) is also commonly called the log return of an asset.

Log returns have some more favourable properties for statistical analysis than the simple net returns \( R_t \).

The following Figure shows the oscillations of the daily logarithmic returns of the stocks considered in this study, although the values plotted were taken in periods of 28 days. As expected, daily returns hover around zero.
Figure 7. Daily Logarithmic Returns of the Assets.
COMPARISON OF PROBABILITY DISTRIBUTIONS OF LOGARITHMIC RETURNS

Next, graphs are generated in which the three frequency distributions proposed in this study appear overlapping:

- The Empirical Distribution curve is generated by simulating 100000 values taken from the sampling of the time series of the real values of the returns of each stock.

- The Normal or Gaussian Distribution curve is generated by simulating 100000 random values with the mean and standard deviation (assuming normality) taken from the time series of the real values.

- The Student’s T Distribution curve is generated by the maximum likelihood estimation (MLE) which is a method of estimating the parameters of a probability distribution by maximizing a likelihood function, so that under the assumed statistical model the observed data is most probable. With the obtained parameters, 100000 values are simulated to produce the curve.

Through visual inspection of these graphs, the assertion that the Scaled t-Distribution fits much better to the empirical distribution of financial returns is compelling. To further support this argument, the fourth statistical moment (the Kurtosis) of the three curves is calculated. The Kurtosis is a metric of the weight of the tails of the distributions. The higher this metric (a value of 3 corresponds to the Normal Distribution), the heavier the tails will be. As can be seen, the Kurtosis of the Student’s T Distribution is always bigger than that of the Normal Distribution.

Figure 8. Comparison of Distributions for Facebook Log Returns.
Figure 9. Comparison of Distributions for Apple Log Returns.

Figure 10. Comparison of Distributions for Amazon Log Returns.
Figure 11. Comparison of Distributions for Netflix Log Returns.

![Graph comparing distributions for Netflix log returns]

Figure 12. Comparison of Distributions for Google Log Returns.

![Graph comparing distributions for Google log returns]
Figure 13. Comparison of Distributions for Tesla Log Returns.

![Graph showing comparison of distributions for Tesla log returns. Empirical, Normal, and Student's T distributions are depicted.](image)

VAR AND ES BASED ON STUDENT’S T DISTRIBUTION

Figure 14. VaR and ES for Facebook.

![Graph showing VaR and ES for Facebook with confidence intervals.](image)
Figure 15. VaR and ES for Apple.

Figure 16. VaR and ES for Amazon.
Figure 17. VaR and ES for Netflix.

Figure 18. VaR and ES for Google.


2 DISCUSSION

This study has shown the superiority of the Scaled Student’s T distribution to model financial phenomena such as time series of the returns of stocks. If the Scaled Student’s T distribution is used regularly for the calculation of VaR and ES, it is possible to arrive at more truthful indicators and therefore more useful for the regulation of trading strategies carried out by portfolio managers, avoiding rogue trading.

Not only financial organizations would benefit from the regular use of the VaR and ES calculated under the Scaled Student’s T distribution, but also individual traders, who would thus have a better estimate of the risk of their investment portfolios.

The usefulness of risk estimators such as VaR and ES calculated under the Scaled Student’s T distribution will avoid severe criticism of these estimators when calculated in a traditional way, such as that sour phrase by David Einhorn:

“Value-at-Risk calculation does not evaluate what happens in the last one percent... This is like an airbag that works all the time, except when you have a car accident.”

The best use of these risk estimators is the one that is executed daily on the returns of shares that make up an investment portfolio. For every investor or portfolio manager, VaR and ES are an unbeatable tool for portfolio re-composition.

Figure 19. VaR and ES for Tesla.
REFERENCES


